# EXCEL OPTIMIZATION SOLVER MODEL DEVELOPMENT FOR OSOGBO'S RAINFALL INTENSITY, DURATION AND FREQUENCY

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## Abstract

The type and velocity of water flow resulting from rainfall activities in a certain catchment are constant variables in hydraulic engineering systems. The recent flooding experienced in Osogbo requires adequate frequency analysis of rainfall record that will aid the design of drainages to mitigate the flooding. The Nigerian Meteorological Agency (NIMET), Abuja, provided twenty-five (25) years' worth of Osogbo daily rainfall data (amount and duration), which were then submitted to frequency analysis for the creation of intensity-duration-frequency models. Using the Excel Optimization Solver wizard, mean rainfall levels with durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 minutes were extracted and subjected to frequency analysis. For return periods of 2, 5, 10, 25, 50, and 100 years, defined and non-specified IDF models were created using the Normal distribution and Pearson type 3 distributions. Osogbo has not seen the development of these models. The fitness of the probability distribution functions was examined using the coefficient of determination ($R^2$) and Mean Squared Error (MSE) values. $R^2$ and MSE values for the normal distribution range from 0.992 to 0.995 and 7.71 to 23.64, while those for the Pearson type 3 distribution range from 0.991 to 0.996 and 5.46 to 33.13. The Ministry of Works can use the developed models for forecast and appropriate design objectives in the city of Osogbo.

**Keywords:** Excel Optimization Solver, Normal distribution, Goodness of fit test, IDF models, Pearson Type 3 distributions, Osogbo.

## Introduction

The first and second Sustainable Development Goals of the United Nations can be achieved by managing water resources effectively in all river basins which is to eradicate poverty and food security; Morton, *et al* (2017). The establishment of Rainfall Intensity Duration Frequency (IDF) connections, which may be utilized for efficient analysis and design of flood control structures, is a result of the use of statistical analysis techniques on rainfall volume and duration. According to (David *et al*., 2019), when there is appropriate understanding of the frequency of severe events, good planning and design actions for extreme events including floods, droughts, high winds, and rainstorms are more likely to be successful. It was stated by (El-Sayed, 2011) that initiatives involving water resources needed to have a solid understanding of IDF (intensity, duration, frequency) modeling. For the frequency study of rainfall amount and duration from rain gauge stations, probability distribution functions are used. The maximum rainfall intensity is the dependent variable, and other relevant factors, which are independent variables, are represented by the IDF formulae, which are empirical equations (for example rainfall duration and frequency). According to (Chow et al, 1988), many of these probability functions are used in practical hydrological applications. Because of its widespread use, accurate assessment of the intensity-duration-frequency relationship has drawn the attention of researchers and scientists from around the world (Mohammed Zakman, 2016). The coverage in Nigeria has expanded from the North Central region to the South-East and South-South, as evidenced by the IDF models in Port Harcourt [Ilaboya & Nwachukwu, 2022; Nwaogazie & Masi, 2019) and Eket in the Awka Ibom State (Nwaogazie & Uba, 2001). These findings support the IDF theory for return periods between two and ten years. It is possible to implement climate smart agriculture practices with adequate IDF knowledge. The management of natural resources (land and water) is essential for removing obstacles faced by small-scale farmers, such as access to technical expertise, poor market access, and insufficient investment (Morton, 2007).

The development of these models is crucial for the appropriate construction of flood mitigation structures to prevent flooding incidents in Osogbo, which were most recently experienced in 2022 and resulted in fatalities and property losses totaling several million of Naira.

## Materials and Methods

### *Study Area Description*

The capital of Osun State, Osogbo located in South – West Nigeria with a distance of about 198 km inland northeast of Lagos and 495 km southwest of Abuja and covering an estimated metropolitan area of about 723 km$^2$. The elevation is 320 m above the sea level and falls within latitude 7$^o$ 33´ 25"N and longitudes 7$^o$ 46´ 15"E. Osogbo lies completely within the tropical forest zone but close to the boundary between the forest and the derived savannah. The mean annual temperature and rainfall of Osogbo metropolis is 28$^0$C & 1,375 mm and it is properly drained by river Osun. The study area is graphically illustrated in Figure 1.

### *Data Collection*

The data, which covered the 25-year period from 1986 to 2010, were provided by the Nigerian Meteorological Agency (NIMET). The data were divided into intervals of five to four hundred twenty minutes. Utilizing the ranking data allowed for the computation of rainfall intensities for the building of various models.

**Figure 1: Map of Osun State showing Osogbo (Fashae *et al.*, 2019)**

### *Data Analysis*

The following durations were chosen to have the largest amount of rainfall: 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 (minutes). Equation (1) illustrates the mathematical expression of the IDF link described by (Chen, 1983; David et al., 2019):

$$I = f(T,d) \qquad (1)$$

Note: intensity is I; return period, T and duration, d.

By dividing the amount by the duration (minutes) and multiplying by 60 as a conversion factor, the rainfall amount is transformed into intensity (mm/hr). For instance, an intensity of (32.6/20) x 60 = 97.8 mm/hr is produced by a rainfall amount of 32.6 mm for a 20-minute period.

Table 1 displays all of Osogbo's intensities for various time frames.

For the station under consideration, rainfall intensity magnitude was determined using a frequency analysis method. The earlier study by (Nwaogazie & Masi, 2019) demonstrated that Pearson Type III and the Normal distribution best suit rainfall models that have been established; as a result, these two probability distribution functions were accepted for computing rainfall intensities for particular return periods.

### *Normal Distribution*

One often utilized probability distribution for determining the values of rainfall intensity is the normal distribution. Equation (2) was used to calculate the numbers for the intensity of the rain:

$$X_T=X+K_TS \qquad (2)$$

Where $X_T$ = rainfall intensity values (magnitude of the hydrologic event)

**Table 1: Ranked Observed Annual Rainfall Intensities (mm/hr) for different Durations (minutes) for Osogbo**

| | Convert to intensity (mm/hr) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | 5 | 10 | 15 | 20 | 30 | 45 | 60 | 90 | 120 | 180 | 240 | 300 | 420 |
| 1 | 192.0 | 138.0 | 106.0 | 97.8 | 73.8 | 54.1 | 45.6 | 39.1 | 29.3 | 21.2 | 17.5 | 15.7 | 14.2 |
| 2 | 187.2 | 121.8 | 102.4 | 96.3 | 73.6 | 52.8 | 45.6 | 33.1 | 28.2 | 20.9 | 16.8 | 14.6 | 13.5 |
| 3 | 178.8 | 121.8 | 100.8 | 85.8 | 73.2 | 50.9 | 42.6 | 32.5 | 28.2 | 19.5 | 16.4 | 14.5 | 12.2 |
| 4 | 175.2 | 111.6 | 100.0 | 84.6 | 71.2 | 49.2 | 42.6 | 32.4 | 27.7 | 19.5 | 15.9 | 14.0 | 11.2 |
| 5 | 172.1 | 108.4 | 92.0 | 84.3 | 70.6 | 48.8 | 42.3 | 30.6 | 26.9 | 18.8 | 15.7 | 13.5 | 11.2 |
| 6 | 171.6 | 100.6 | 90.4 | 83.4 | 68.6 | 47.5 | 42.0 | 28.4 | 26.8 | 18.5 | 15.6 | 13.1 | 10.4 |
| 7 | 159.7 | 96.0 | 82.7 | 75.6 | 64.8 | 45.7 | 38.2 | 28.4 | 26.7 | 18.4 | 15.1 | 13.0 | 10.2 |
| 8 | 134.6 | 93.6 | 76.8 | 75.3 | 64.2 | 44.8 | 36.9 | 28.0 | 26.3 | 17.9 | 15.1 | 12.7 | 10.0 |
| 9 | 132.6 | 89.4 | 75.2 | 71.1 | 63.6 | 39.8 | 36.6 | 25.1 | 26.2 | 17.9 | 14.7 | 12.6 | 9.6 |
| 10 | 129.4 | 84.8 | 64.7 | 68.3 | 56.4 | 36.9 | 35.8 | 23.9 | 26.2 | 17.8 | 13.9 | 12.0 | 9.4 |
| 11 | 129.2 | 84.0 | 63.8 | 63.4 | 55.6 | 34.8 | 35.6 | 23.3 | 26.2 | 17.5 | 13.8 | 11.7 | 9.3 |
| 12 | 126.8 | 83.6 | 62.4 | 60.0 | 52.2 | 31.1 | 34.3 | 19.6 | 26.0 | 17.4 | 13.4 | 11.4 | 9.1 |
| 13 | 122.3 | 81.5 | 62.2 | 53.4 | 52.1 | 30.7 | 32.8 | 19.3 | 24.4 | 17.4 | 13.3 | 11.2 | 9.0 |
| 14 | 119.2 | 81.4 | 62.1 | 52.6 | 48.4 | 29.9 | 30.5 | 18.8 | 24.3 | 17.3 | 13.2 | 11.1 | 9.0 |
| 15 | 116.7 | 79.9 | 61.0 | 51.3 | 40.8 | 29.9 | 25.7 | 18.8 | 21.0 | 16.8 | 13.1 | 11.1 | 8.9 |
| 16 | 114.1 | 77.0 | 59.6 | 51.3 | 40.2 | 29.3 | 25.3 | 18.5 | 20.7 | 16.2 | 13.0 | 10.7 | 8.6 |
| 17 | 99.9 | 75.1 | 59.2 | 50.3 | 39.2 | 28.3 | 24.7 | 17.8 | 19.2 | 16.2 | 13.0 | 10.7 | 8.6 |
| 18 | 97.8 | 73.5 | 58.8 | 48.5 | 39.1 | 27.6 | 24.6 | 17.4 | 17.9 | 15.8 | 12.6 | 10.5 | 8.4 |
| 19 | 95.6 | 71.9 | 57.3 | 47.3 | 38.4 | 27.0 | 24.2 | 17.0 | 16.2 | 14.7 | 12.2 | 10.5 | 8.3 |
| 20 | 88.3 | 63.0 | 56.1 | 46.8 | 37.0 | 26.4 | 23.3 | 16.6 | 15.9 | 14.0 | 12.2 | 10.4 | 8.2 |
| 21 | 44.4 | 61.6 | 54.9 | 46.3 | 36.1 | 23.1 | 22.7 | 14.6 | 15.5 | 12.4 | 12.1 | 10.4 | 7.9 |
| 22 | 42.0 | 60.6 | 48.0 | 45.3 | 35.3 | 22.6 | 22.3 | 14.2 | 15.5 | 12.2 | 10.5 | 9.7 | 7.7 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 30.0 | 60.2 | 47.0 | 39.7 | 34.6 | 22.1 | 21.8 | 13.9 | 15.2 | 11.9 | 10.2 | 9.7 | 7.6 |
| 24 | 28.8 | 55.6 | 46.0 | 38.8 | 30.3 | 20.4 | 19.1 | 12.9 | 14.7 | 11.9 | 10.0 | 8.8 | 7.5 |
| 25 | 19.2 | 21.0 | 42.5 | 37.9 | 29.6 | 20.0 | 18.7 | 12.7 | 14.3 | 11.6 | 9.8 | 8.7 | 7.5 |
| Mean | 116.3 | 83.8 | 69.3 | 62.2 | 51.6 | 34.9 | 31.8 | 22.3 | 22.4 | 16.6 | 13.6 | 11.7 | 9.5 |
| Standard Deviation | 51.8 | 24.9 | 19.3 | 18.6 | 15.4 | 11.1 | 8.8 | 7.4 | 5.3 | 2.9 | 2.1 | 1.8 | 1.8 |
| Coefficient of Skewness | 0.30 | -0.02 | 0.67 | 0.51 | 0.16 | 0.40 | 0.11 | 0.59 | -0.37 | -0.46 | -0.08 | 0.42 | 1.24 |

The model parameters C, m, and a, were obtained by calibrating the non-linear power law given by Equation (4) using the Excel optimization solver. Equation (3) is the standard Normal distribution frequency factor Equation in Hydrology.

$$K_T = w - \frac{2.515517+0.802853w+0.010328w^2}{1+ 1.432788w + 0.189269w^2+0.001308w^2} \qquad (3)$$

Where w = Intermediate Variable and is given in Equation (3a) as:

$$w = \left[In\left(\frac{1}{P^2}\right)\right]^{1/2} \qquad (3a)$$

And P = exceedance probability given in Equation (3b) as:

$$P = \frac{1}{T} \qquad (3b)$$

Where T = return period

Example: Normal distribution frequency factor for a 5 years return period

$P = \frac{1}{5} = 0.2$

$w = \left[In\left(\frac{1}{0.2^2}\right)\right]^{1/2} = 1.794$

Substituting our values into Equation (3) we have;

$K_T = w - \frac{2.515517+0.802853(1.794)+0.010328(1.794)^2}{1+ 1.432788(1.794) + 0.189269(1.794)^2+0.001308(1.794)^3}$

$K_T = 0.841457$

Table 2 displays the calculated $K_T$ values for the Normal distribution for various return durations.

Table 2: Normal distribution frequency factor

| Return Period | 2 | 5 | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|---|
| P | 0.5 | 0.2 | 0.1 | 0.04 | 0.02 | 0.01 |
| W | 1.17741 | 1.794123 | 2.145966 | 2.537272 | 2.79715 | 3.034854 |
| $K_T$ values | -1E-07 | 0.841457 | 1.281729 | 1.751077 | 2.054189 | 2.326785 |

### E. IDF Model Calibration

Sherman's IDF model is given as (4)

$$I = \frac{CT_r^m}{T_d^e} \qquad (4)$$

I = rainfall intensity, C, m and a = model parameters, Tr = return period (year) and Td = duration (hours)

Excel optimization solver was used to calibrate the non-linear power law given by Equation (4) to obtain the model parameters C, m and a.

### *Goodness of Fit Test*

Using the Anderson-Darling test, the Normal distribution and Pearson Type 3 fit the rainfall intensities with 0.711 and 0.754 significant values at the 5% confidence level, respectively.

## Results and Discussion

The Normal distribution model best fits the highest rainfall amounts as indicated in Table 3 according to the values for the estimated coefficient of determination, $R^2$, and Mean Square Error for a particular return period.

The values for rainfall intensity were computed using Equation (1). The mean and standard deviation of the Normal distribution of rainfall intensity are seen in Table 1. Equation (2) is used to compute the probability equivalent of rainfall intensity using Normal distribution for a duration of 10 minutes and a return period of 5 years by substituting the values of $X_T$, $K_T$, and S from Table 1.

Using the Anderson-Darling test, the Normal distribution and Pearson Type 3 fit the rainfall intensities with 0.711 and 0.754 significant values at the 5% confidence level, respectively.

$X_T$ = 116.3 + (0.841x 51.8)) = 159.86 mm/hr

### *Specified Return Period IDF Models Calibration*

The calibration of Sherman equation IDF models for given return durations was shown in (David et al., 2019). The results for the Normal distribution are shown in Table 3, together with the mean square error (MSE) and $R^2$ coefficients of determination.

Model parameters C, m, and a, are generated for a particular duration and return period after Equation (4) is calibrated using Excel optimization software. These IDF models are return period specific, as opposed to the non-specific models that are shown in Table 3. (see Equation 7).

**Table 3: Developed IDF Models for different return periods using Normal Distribution rainfall intensity values for Osogbo**

| Return Period | IDF Model ± | Coefficient of Determination($R^2$) | MSE |
|---|---|---|---|
| 2 | $I = \frac{T_r^{5.617}}{0.527}$ | 0.992 | 7.71 |
| 5 | $I = \frac{T_r^{3.264}}{0.569}$ | 0.995 | 8.26 |
| 10 | $I = \frac{T_r^{2.468}}{0.585}$ | 0.995 | 10.98 |
| 25 | $I = \frac{T_r^{1.879}}{0.599}$ | 0.995 | 15.60 |
| 50 | $I = \frac{T_r^{1.593}}{0.607}$ | 0.994 | 19.51 |
| 100 | $I = \frac{T_r^{1.663}}{0.776}$ | 0.996 | 132.47 |

± return period specific IDF models

### *Evaluation of iterative Equation Solver in Excel*

The Excel Solver software was used to evaluate the model's parameters over a 100-year defined return period. The generic IDF model presented in Equation has undergone ten (10) iterations (7). The tabular computation of coefficient of determination is shown in Table 4 while Table 5 shows the iterative values of C, m and a with the use of Excel Solver on data in Table 4.

**Table 4: Tabular Computation of Coefficient of Determination for Osogbo**

| Intensity, I | Predicted Intensity, $I_p$ | $(I-I_p)^2$ | $(I-I_{avg})^2$ |
|---|---|---|---|
| 702.2254 | 692.1387 | 101.7412 | 276600.1 |
| 373.1987 | 404.0189 | 949.8814 | 38770.07 |
| 295.9517 | 294.878 | 1.152859 | 14317.12 |
| 253.6446 | 235.836 | 317.1467 | 5982.571 |
| 164.8572 | 172.1277 | 52.86084 | 130.8817 |
| 139.5848 | 125.6295 | 194.7521 | 1347.822 |
| 107.553 | 100.4753 | 50.09349 | 4725.815 |
| 74.33462 | 73.3331 | 1.003031 | 10396.44 |
| 57.04242 | 58.64997 | 2.58421 | 14221.78 |
| 37.21077 | 42.80638 | 31.31084 | 19345.13 |
| 32.40079 | 34.23547 | 3.366051 | 20706.27 |
| 27.7916 | 28.78805 | 0.992918 | 22054.02 |
| 26.07234 | 22.16785 | 15.24509 | 22567.61 |
| 176.2975 | | 1722.131 | 451165.7 |

**Table 5: Model parameters for Sherman's specific IDF model calibration**

| Iteration | C | M | a |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1.08689 | 1.4 | 0.824064 |
| 3 | 1.129055 | 1.611199 | 0.727277 |
| 4 | 1.132479 | 1.635577 | 0.721812 |
| 5 | 1.137515 | 1.662717 | 0.772641 |
| 6 | 1.137736 | 1.663208 | 0.776114 |
| 7 | 1.137797 | 1.663488 | 0.77664 |
| 8 | 1.137797 | 1.663488 | 0.776639 |
| 9 | 1.137797 | 1.663488 | 0.776639 |
| 10 | 1.137797 | 1.663488 | 0.776639 |

A General IDF model was also developed. A total of 13 durations multiplied by 6 return period yields 78 input data point. The entire input data were taken from Table 1.

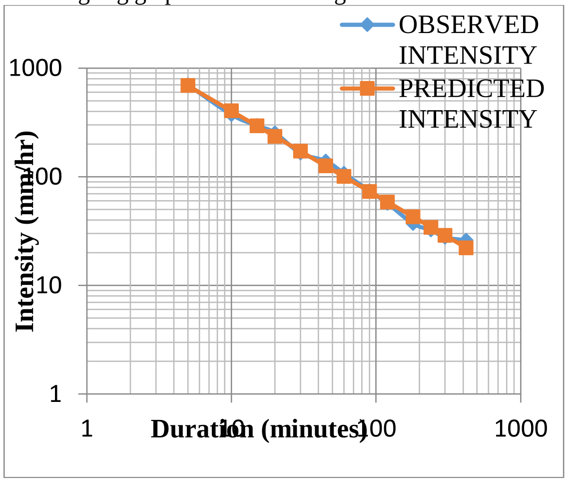
Programmed least squares equations were used to develop a general (non-specified) IDF model using Excel Optimization Solver. This resulted in Equation (7).

$$I = \frac{330.98 T_r^{\,0.135}}{T_d^{\,0.592}} \qquad (7)$$

Coefficient of determinant ($R^2$) = 0.989; Mean Squared Error = 29.23 mm/hr

***Comparison of Observed and Predicted Rainfall Intensities***

This model enables one to predict the intensity of rainfall of any duration and any return period. The verification of the developed model is carried out by plotting the observed and predicted intensities on the same log-log graph as shown in Figures 2.



**Figures 2: Log-Log comparative plot for observed and predicted intensities**

**Comparison of Regression Approach and Excel** Optimization Solver results for model parameters, $R^2$ and MSE

Table 6 (an extension of Table 5) clearly shows the result from Excel Optimization Solver option is more reliable than the normal regression method, the conventional simultaneous solution using matrix i.e. Gauss elimination, inverse or determinant approach.

**Table 6: Results from Regression Approach and Excel Solver Optimization Approach (Normal distribution, 100 year return period)**

| Method | C | m | A | $R^2$ | MSE |
|---|---|---|---|---|---|
| Regression | 65.42 | 3.533 | 0.575 | 0.865 | 324.40 |
| Excel solver | 1.138 | 1.663 | 0.776 | 0.996 | 132.47 |

**Conclusion**

The trend of higher intensities occurring at lower duration which is found in literature has been observed in the developed models for Normal distribution and Pearson Type 3 distributions. The prediction of rainfall intensity with the PDFs showed a good match with observed intensity values. The Normal distribution model ranked as the best with respect to MSE 23.64 and $R^2$ 0.995 in the return period specific model. The developed models can be used to obtain design intensity for drainage design for effective flood control.

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